# Aiming for Grade 7 REVISION BOOKLET 

Name: $\qquad$


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## Surds

## Things to remember:

- $\sqrt{ }$ means square root;
- To simplify surds, find all its factors;
- To rationalise the denominator, find an equivalent fraction where the denominator is rational.


## Questions:

1. Work out
$\frac{(5+\sqrt{3})(5-\sqrt{3})}{\sqrt{22}}$
Give your answer in its simplest form.
(Total 3 marks)
2. (a) Rationalise the denominator of $\frac{1}{\sqrt{3}}$
(b) Expand $(2+\sqrt{3})(1+\sqrt{3})$

Give your answer in the form $a+b \sqrt{3}$ where $a$ and $b$ are integers.
3. (a) Rationalise the denominator of $\frac{1}{\sqrt{7}}$
(b) (i) Expand and simplify $(\sqrt{3}+\sqrt{15})^{2}$

Give your answer in the form $a+b \sqrt{3}$ where $a$ and $b$ are integers.
(ii) All measurements on the triangle are in centimetres. $A B C$ is a right-angled triangle. $k$ is a positive integer.

Diagram NOT
accurately drawn

Find the value of $k$.

$$
k=
$$

4. Expand and simplify $(\sqrt{3}-\sqrt{2})(\sqrt{3}-\sqrt{2})$
5. (a) Write down the value of $49^{1 / 2}$
(b) Write $\sqrt{45}$ in the form $k \sqrt{5}$, where $k$ is an integer.
6. Write $\frac{\sqrt{18}+10}{\sqrt{2}}$ in the form $a+b \sqrt{3}$ where $a$ and $b$ are integers.

$$
\begin{aligned}
& a= \\
& b=
\end{aligned}
$$

7. Expand and simplify $(2+\sqrt{3})(7-\sqrt{3})$

Give your answer in the form $a+b \sqrt{3}$ where $a$ and $b$ are integers.
8. Rationalise the denominator of $\frac{(4+\sqrt{2})(4-\sqrt{2)}}{\sqrt{7}}$

Give your answer in its simplest form.

## (Total for question = 3 marks)

9. Show that $\frac{(4-\sqrt{3})(4+\sqrt{3})}{\sqrt{13}}$ simplifies to $\sqrt{13}$

## Bounds Calculations

## Things to remember:

- Calculating bounds is the opposite of rounding - they are the limits at which you would round up instead of down, and vice versa.
- When dividing bounds, $\mathrm{UB}=\mathrm{UB} \div \mathrm{LB}$ and $\mathrm{LB}=\mathrm{LB} \div \mathrm{UB}$


## Questions:

1. A piece of wood has a length of 65 centimetres to the nearest centimetre.
(a) What is the least possible length of the piece of wood?
(b) What is the greatest possible length of the piece of wood?
2. Chelsea's height is 168 cm to the nearest cm .
(a) What is Chelsea's minimum possible height?
(b) What is Chelsea's maximum possible height?
3. $\quad I=\frac{V}{R}$
$V=250$ correct to the nearest 5
$R=3900$ correct to the nearest 100
Work out the lower bound for the value of $I$.
Give your answer correct to 3 decimal places.
You must show your working.
4. Here is a solid bar made of metal.

The bar is in the shape of a cuboid.
The height of the bar is $h \mathrm{~cm}$.
The base of the bar is a square of side $d \mathrm{~cm}$.
The mass of the bar is $M \mathrm{~kg}$.
$d=8.3$ correct to 1 decimal place.
$M=13.91$ correct to 2 decimal places.
$h=84$ correct to the nearest whole number.
Find the value of the density of the metal to an appropriate degree of accuracy.
Give your answer in $\mathrm{g} / \mathrm{cm}^{3}$.
You must explain why your answer is to an appropriate degree of accuracy.

(Total for question = 5 marks)
5. Steve travelled from Ashton to Barnfield.

He travelled 235 miles, correct to the nearest 5 miles.
The journey took him 200 minutes, correct to the nearest 5 minutes.
Calculate the lower bound for the average speed of the journey.
Give your answer in miles per hour, correct to 3 significant figures.
You must show all your working.
6. The value of $p$ is 4.3

The value of $q$ is 0.4
Both $p$ and $q$ are given correct to the nearest 0.1
(a) Write down the lower bound for $p$.
$r=p+\frac{1}{q}$
(b) Work out the upper bound for $r$.

You must show all your working.

$$
m=\frac{\sqrt{s}}{t} \quad s=3.47 \text { correct to } 3 \text { significant figures }
$$

7. 

$$
t=8.132 \text { correct to } 4 \text { significant figures }
$$

By considering bounds, work out the value of $m$ to a suitable degree of accuracy. Give a reason for your answer.
8. $a$ is 8.3 cm correct to the nearest mm $b$ is 6.1 cm correct to the nearest mm


Calculate the upper bound for $c$.
You must show your working.

## Parallel and Perpendicular Graphs

Things to remember:

- The general equation of a linear graph is given by $y=m x+c$, where $m$ is the gradient and $c$ is the y-intercept.
- Parallel graphs have the same gradient.
- Gradients of perpendicular graphs have a product of -1.


## Questions:

1. The diagram shows a straight line, $L_{1}$, drawn on a grid.


A straight line, $L_{2}$, is parallel to the straight line $L_{1}$ and passes through the point $(0,-5)$. Find an equation of the straight line $L_{2}$.
2. The straight line $\mathbf{L}$ has equation $y=2 x-5$

Find an equation of the straight line perpendicular to $L$ which passes through ( $-2,3$ ).
3. In the diagram, $A B C$ is the line with equation $y=-\frac{1}{2} x+5$
$A B=B C$
$D$ is the point with coordinates $(-13,0)$


Find an equation of the line through $A$ and $D$.
4. Here are the graphs of 6 straight lines.

Graph A


Graph C


Graph E


Graph B


Graph D


Graph $\mathbf{F}$


Match each of the graphs $\mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{D}, \mathbf{E}$ and $\mathbf{F}$ to the equations in the table.

| Equation | $y=\frac{1}{2} x+2$ | $y=2 x-2$ | $y=-\frac{1}{2} x+2$ | $y=-2 x-2$ | $y=2 x+2$ | $y=-\frac{1}{2} x-2$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Graph |  |  |  |  |  |  |

5. In the diagram, $\quad A$ is the point $(-2,0)$
$B$ is the point $(0,4)$
C is the point $(5,-1)$


Find an equation of the line that passes through $C$ and is perpendicular to $A B$.
(Total for Question is 4 marks)
6. Find an equation of the straight line that is perpendicular to the straight line $x+2 y=5$ and that passes through the point $(3,7)$.
*7. $\quad \mathbf{A}$ and $\mathbf{B}$ are straight lines.
Line $\mathbf{A}$ has equation $2 y=3 x+8$
Line $\mathbf{B}$ goes through the points $(-1,2)$ and $(2,8)$
Do lines $\mathbf{A}$ and $\mathbf{B}$ intersect?
You must show all your working.
(Total for Question is 3 marks)
8. A straight line, $L$, is perpendicular to the line with equation $y=1-3 x$. The point with coordinates $(6,3)$ is on the line $L$.
Find an equation of the line $\boldsymbol{L}$.

## Transformations of graphs

## Things to remember:

- $f(x)$ means the function of $x$.
- $-f(x)$ is a reflection in the $x$-axis.
- $f(-x)$ is a reflection in the $y$-axis.
- $f(x-a)$ is a translation in the $x$-axis, a units.
- $f(x)+b$ is a translation in the $y$-axis, $b$ units.
- $\quad c f(x)$ is an enlargement in the $y$-axis, scale factor $c$.
- $f(d x)$ is an enlargement in the $x$-axis, scale factor $\frac{1}{d}$.


## Questions:

1. $y=\mathrm{f}(x)$

The graph of $y=\mathrm{f}(x)$ is shown on the grid.

(a) On the grid above, sketch the graph of $y=-\mathrm{f}(x)$.

The graph of $y=\mathrm{f}(x)$ is shown on the grid.


The graph $\mathbf{G}$ is a translation of the graph of $y=\mathrm{f}(x)$.
(b) Write down the equation of graph $\mathbf{G}$.
2. The graph of $y=\mathrm{f}(x)$ is shown on both grids below.

(a) On the grid above, sketch the graph of $y=\mathrm{f}(-\mathrm{x})$

(b) On this grid, sketch the graph of $y=-\mathrm{f}(x)+3$
3. The graph of $y=\mathrm{f}(x)$ is shown on each of the grids.
(a) On this grid, sketch the graph of $y=\mathrm{f}(x-3)$

(b) On this grid, sketch the graph of $y=2 f(x)$

4. The graph of $y=\mathrm{f}(x)$ is shown on the grid.

(a) On the grid above, sketch the graph of $y=\mathrm{f}(x+3)$

The graph of $y=\mathrm{g}(x)$ is shown below.


The graph $\mathbf{G}$ is the reflection of $y=g(x)$ in the $x$-axis.
(b) Write down an equation of graph $\mathbf{G}$.

## Algebraic Fractions - Simplifying

Things to remember:

- Factorise the numerator and denominator;
- Cancel common factors;
- Then add/subtract/multiply divide if necessary.


## Questions:

1. Simplify $\frac{p^{2}-9}{2 p+6}$
2. Simplify fully $\frac{6 x^{2}+3 x}{4 x^{2}-1}$
3. Simplify $\frac{x^{2}+2 x+1}{x^{2}+3 x+2}$
4. Simplify fully $\frac{x^{2}+x-6}{x^{2}-7 x+10}$
5. Simplify fully $\frac{x^{2}-8 x+15}{2 x^{2}-7 x-15}$
6. Simplify fully $\frac{2 x^{2}+3 x+1}{x^{2}-3 x-4}$
7. (a) Simplify $\frac{2 x+4}{x^{2}+4 x+4}$
(b) Write $\frac{1}{x+4}+\frac{2}{x-4}$ as a single fraction in its simplest form.
8. Simplify fully $\frac{x+3}{4}+\frac{x-5}{3}$

## Algebraic fractions - solving

Things to remember:

- Multiply every term by the product of the denominators;
- Solve to find x .


## Questions:

1. Solve $\frac{5(2 x+1)}{3}=4 x+7$

$$
x=
$$

$\qquad$
2. (a) Show that the equation $\frac{5}{x+2}=\frac{4-3 x}{x-1}$
can be rearranged to give $3 x^{2}+7 x-13=0$
(b) Solve $3 x^{2}+7 x-13=0$

Give your solutions correct to 2 decimal places.

"
$\qquad$
3. Solve the equation $\frac{x}{2 x-3}+\frac{4}{x+1}=1$

$$
x=
$$

(Total 5 marks)
4. Solve the equation $\frac{3}{x+3}-\frac{4}{x-3}=\frac{5 x}{x^{2}-9}$

$$
x=
$$

5. (a) Solve $\frac{3}{x}+\frac{3}{2 x}=2$

$$
x=. .
$$

(b) Using your answer to part (a), or otherwise, solve $\frac{3}{(y-1)^{2}}+\frac{3}{2(y-1)^{2}}=2$

$$
y=.
$$

## Solving Quadratic Inequalities

Things to remember:

- Start by solving the quadratic to find the values of $x$, then sketch the graph to determine the inequality.


## Questions:

1. Solve $x^{2}>3 x+4$
2. Solve the inequality $x^{2}>3(x+6)$
3. Solve the inequality $x^{2}+5 x>6$
4. 

Solve the inequality
$x^{2}-2 x-8<0$
(Total for question = 3 marks)
5. Solve the inequality $x^{2}-x \geq 12$
6. Solve the inequality $x^{2} \leq 4(2 x+5)$

## Circle theorems

Things to remember:


The angle at the centre is twice the angle at the circumference.


The angle in a semicircle is $90^{\circ}$


Angles subtended by the same arc are equal.


Opposite angles in a cyclic quadrilateral sum to $180^{\circ}$


Angles in alternate segments are equal.

## Questions:

1. 



Diagram NOT accurately drawn
$P$ is a point on the circumference of the circle, centre $O$.
$P Q$ is a tangent to the circle.
(i) Write down the size of angle $O P Q$.
(ii) Give a reason for your answer.
$\qquad$
$\qquad$
2.


## Diagram NOT accurately drawn

$A, B$ and $C$ are points on the circumference of a circle, centre $O$.
$A C$ is a diameter of the circle.
(a) (i) Write down the size of angle $A B C$.
(ii) Give a reason for your answer.
$\qquad$
$\qquad$


Diagram NOT accurately drawn
$D, E$ and $F$ are points on the circumference of a circle, centre $O$.
Angle $D O F=130^{\circ}$.
(b) (i) Work out the size of angle $D E F$.
(ii) Give a reason for your answer.
$\qquad$
$\qquad$
3.


Diagram NOT accurately drawn
$A$ and $B$ are points on the circumference of a circle, centre $O$.
$P A$ and $P B$ are tangents to the circle.
Angle $A P B$ is $86^{\circ}$.
Work out the size of the angle marked $x$.
$\qquad$
4.


Diagram NOT accurately drawn
In the diagram, $A, B, C$ and $D$ are points on the circumference of a circle, centre $O$.
Angle $B A D=70^{\circ}$.
Angle $B O D=x^{\circ}$.
Angle $B C D=y^{\circ}$.
(a) (i) Work out the value of $x$.
(ii) Give a reason for your answer.
$\qquad$
$\qquad$
(b) (i) Work out the value of $y$.
$\qquad$
(ii) Give a reason for your answer.
$\qquad$
$\qquad$
5.


Diagram NOT accurately drawn
The diagram shows a circle centre $O$.
$A, B$ and $C$ are points on the circumference.
$D C O$ is a straight line.
$D A$ is a tangent to the circle.
Angle $A D O=36^{\circ}$
(a) Work out the size of angle $A O D$.
(b) (i) Work out the size of angle $A B C$.
(ii) Give a reason for your answer.
$\qquad$
$\qquad$

## Vectors

## Things to remember:

- Use the letter provided in the question.
- Going against the arrow is a negative.
- Vectors need to be written in bold or underlined.
- They can be manipulated similarly to algebra.


## Questions:

1. The diagram shows a regular hexagon $A B C D E F$ with centre $O$.

$\overrightarrow{O A}=6 \mathbf{a} \quad \overrightarrow{O B}=6 \mathbf{b}$
(a) Express in terms of $\mathbf{a}$ and/or $\mathbf{b}$
(i) $\overrightarrow{A B}$,
(ii) $\overrightarrow{E F}$.
$X$ is the midpoint of $B C$.
(b) Express $\overrightarrow{E X}$ in terms of $\mathbf{a}$ and/or $\mathbf{b}$
$Y$ is the point on $A B$ extended, such that $A B: B Y=3: 2$
(c) Prove that $E, X$ and $Y$ lie on the same straight line.
2. $\quad T$ is the point on $P Q$ for which $P T: T Q=2: 1$.

$O P Q$ is a triangle.
$\overrightarrow{O P}=\mathbf{a}$ and $\overrightarrow{O Q}=\mathbf{b}$.
(a) Write down, in terms of $\mathbf{a}$ and $\mathbf{b}$, an expression for ${ }^{\overrightarrow{P Q}}$.

$$
\overrightarrow{P Q}=
$$

(b) Express $\overrightarrow{O T}$ in terms of $\mathbf{a}$ and $\mathbf{b}$.

Give your answer in its simplest form.

$$
\overrightarrow{O T}=
$$

3. $O A B C$ is a parallelogram.


Diagram NOT
accurately drawn
$P$ is the point on $A C$ such that $A P={ }^{\frac{2}{3}} A C$.
$\overrightarrow{O A}=6 \mathbf{a} \cdot \overrightarrow{O C}=6 \mathbf{c}$.
(a) Find the vector $\overrightarrow{O P}$.

Give your answer in terms of $\mathbf{a}$ and $\mathbf{c}$.

The midpoint of $C B$ is $M$.
(b) Prove that $O P M$ is a straight line.
4. $\quad O P Q$ is a triangle.
$R$ is the midpoint of $O P$.
$S$ is the midpoint of $P Q$.
$\overrightarrow{O P}=\mathbf{p}$ and $\overrightarrow{O Q}=\mathbf{q}$


Diagram NOT
accurately drawn
(i) Find $\overrightarrow{o s}$ in terms of $\mathbf{p}$ and $\mathbf{q}$.

$$
\overrightarrow{O S}=
$$

$\qquad$
5. $O P Q R$ is a trapezium with $P Q$ parallel to $O R$.
$\overrightarrow{O P}=2 \mathbf{b} \quad \overrightarrow{P Q}=2 \mathbf{a} \quad \overrightarrow{O R}=6 \mathbf{a}$
$M$ is the midpoint of $P Q$ and $N$ is the midpoint of $O R$.

(a) Find the vector $\overrightarrow{M N}$ in terms of $\mathbf{a}$ and $\mathbf{b}$.

$$
\overrightarrow{M N}=
$$

$X$ is the midpoint of $M N$ and $Y$ is the midpoint of $Q R$.
(b) Prove that $X Y$ is parallel to $O R$.
6. $A B C D$ is a straight line.

$O$ is a point so that $\overrightarrow{O A}=\mathbf{a}$ and $\overrightarrow{O B}=\mathbf{b}$. $B$ is the midpoint of $A C$.
$C$ is the midpoint of $A D$.
Express, in terms of $\mathbf{a}$ and $\mathbf{b}$, the vectors (i) $\overrightarrow{A C}$
(ii) $\xrightarrow{O D}$
7. Diagram NOT accurately drawn

$A B C D$ is a parallelogram.
$A B$ is parallel to $D C$.
$A D$ is parallel to $B C$.
$\overrightarrow{A B}=\mathbf{p}$
$\overrightarrow{A D}=\mathbf{q}$
(a) Express, in terms of p and q
(i) $\overrightarrow{A C}$
(ii) $\overrightarrow{B D}$

Diagram NOT accurately drawn

$A C$ and $B D$ are diagonals of parallelogram $A B C D$.
$A C$ and $B D$ intersect at $T$.
(b) Express $\overrightarrow{A T}$ in terms of $\mathbf{p}$ and $\mathbf{q}$.
8. Diagram NOT accurately drawn
$O A B$ is a triangle.
$B$ is the midpoint of $O R$.
$Q$ is the midpoint of $A B$.
$\overrightarrow{O P}=2 \mathbf{a} \quad \overrightarrow{P A}=\mathbf{a} \quad \overrightarrow{O B}=\mathbf{b}$

(a) Find, in terms of $\mathbf{a}$ and $\mathbf{b}$, the vectors
(i) $\overrightarrow{A B}$,
(ii) $\overrightarrow{P R}$,
(iii) $\overrightarrow{P Q}$.
(b) Hence explain why $P Q R$ is a straight line.

The length of $P Q$ is 3 cm .
(c) Find the length of $P R$.

## Sine and Cosine Rules

Things to remember:

- For any triangle $A B C, a^{2}=b^{2}+c^{2}-2 b c \cos A$
- For any triangle $\mathrm{ABC}, \frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$
- For any triangle $A B C$, are $a=1 / 2 a b \sin C$


## Questions:



1. Diagram NOT accurately drawn $A B C$ is a triangle.
$D$ is a point on $A C$.
Angle $B A D=45^{\circ}$
Angle $A D B=80^{\circ}$
$A B=7.4 \mathrm{~cm}$
$D C=5.8 \mathrm{~cm}$
Work out the length of $B C$.
Give your answer correct to 3
significant figures.

2. Diagram NOT accurately drawn
$A B C$ is a triangle.
$A B=8.7 \mathrm{~cm}$.
Angle $A B C=49^{\circ}$.
Angle $A C B=64^{\circ}$.
Calculate the area of triangle $A B C$.
Give your answer correct to 3 significant figures.

3. $A B C D$ is a quadrilateral.

Diagram NOT accurately drawn
Work out the length of $D C$.
Give your answer correct to 3 significant figures.

4. Diagram NOT accurately drawn $A B C$ is an isosceles triangle.
Work out the area of the triangle.
Give your answer correct to 3 significant figures.

5. Diagram NOT accurately drawn The diagram shows triangle $L M N$. Calculate the length of $L N$. Give your answer correct to 3 significant figures.

6. $\quad V A B C D$ is a solid pyramid.
$A B C D$ is a square of side 20 cm .
The angle between any sloping edge and the plane $A B C D$ is $55^{\circ}$
Calculate the surface area of the pyramid.
Give your answer correct to 2 significant figures.

7. The diagram shows triangle $A B C$. The area of triangle $A B C$ is $k \sqrt{ } 3$ $\mathrm{cm}^{2}$.
Find the exact value of $k$.


$$
\mathrm{k}=
$$

$\qquad$
(Total for question = 7 marks)
8. Diagram NOT accurately drawn
$A C=9.2 \mathrm{~m}$
$B C=14.6 \mathrm{~m}$
Angle $A C B=64^{\circ}$
(a) Calculate the area of the triangle $A B C$.
Give your answer correct to 3 significant figures.

$\mathrm{m}^{2}$
(b) Calculate the length of $A B$.

Give your answer correct to 3 significant figures.

## Cumulative frequency and box plots

## Things to remember:

- Use a running total - adding on to complete the cumulative frequency column;
- Plot at the end of the group;
- Join up with a smooth curve;
- To find the median find the value half way down the cumulative frequency, draw across to the line and then vertically down to find the value - always show these working lines;
- To find the interquartile range find the upper quartile and the lower quartile and subtract them.
- To draw a box plot
- When comparing box plots, use the median and the IQR and keep words consistent with the question.



## Questions:

1. The table shows information about the heights of 40 bushes.

| Height (h cm) | Frequency | Cumulative Frequency |
| :--- | :--- | :--- |
| $170 \leq h<175$ | 5 |  |
| $175 \leq h<180$ | 18 |  |
| $180 \leq h<185$ | 12 |  |
| $185 \leq h<190$ | 4 |  |
| $190 \leq h<195$ | 1 |  |

(a) Complete the cumulative frequency table above.
(b) On the grid, draw a cumulative frequency graph for your table.

2. The table gives information about the ages of 160 employees of an IT company.

| Age ( $\boldsymbol{A}$ ) in years | Frequency | Cumulative Frequency |
| :--- | :--- | :--- |
| $15<A \leq 25$ | 44 |  |
| $25<A \leq 35$ | 56 |  |
| $35<A \leq 45$ | 34 |  |
| $45<A \leq 55$ | 19 |  |
| $55<A \leq 65$ | 7 |  |

(a) Write down the modal class interval.
(b) Complete the cumulative frequency table.
(c) On the grid below, draw a cumulative frequency graph for your table.

(d) Use your graph to find an estimate for
(i) the median age of the employees,
(i) the interquartile range of the ages of the employees.

Another IT company has 80 employees. The age of the youngest employee is 24 years. The age of the oldest employee is 54 years. The median age is 38 years. The lower quartile age is 30 years. The upper quartile age is 44 years.
(e) On the grid below, draw a box plot to show information about the ages of the employees.

(Total 9 marks)
3. A company tested 100 batteries. The table shows information about the number of hours that the batteries lasted.

| Time ( $\boldsymbol{t}$ hours) | Frequency | Cumulative Frequency |
| :--- | :--- | :--- |
| $50 \leq t<55$ | 12 |  |
| $55 \leq t<60$ | 21 |  |
| $60 \leq t<65$ | 36 |  |
| $65 \leq t<70$ | 23 |  |
| $70 \leq t<75$ | 8 |  |

(a) Complete the cumulative frequency table for this information.
(b) On the grid, draw a cumulative frequency graph for your completed table.

(c) Use your completed graph to find an estimate for the median time. You must state the units of your answer.
4. The table shows information about the ages of the 240 people at a club.

| Age ( $\boldsymbol{t}$ years) | Frequency | Cumulative Frequency |
| :--- | :--- | :--- |
| $15 \leq t<20$ | 95 |  |
| $20 \leq t<25$ | 90 |  |
| $25 \leq t<30$ | 35 |  |
| $30 \leq t<35$ | 15 |  |
| $35 \leq t<40$ | 5 |  |

(a) Complete the cumulative frequency table.
(b) On the grid, draw the cumulative frequency graph for your table.

(c) Use your graph to find an estimate for the median age of the people.
5. An operator took 100 calls at a call centre. The table gives information about the time ( $t$ seconds) it took the operator to answer each call.

| Time (t seconds) | Frequency | Cumulative Frequency |
| :--- | :--- | :--- |
| $0<t \leq 10$ | 16 |  |
| $10<t \leq 20$ | 34 |  |
| $20<t \leq 30$ | 32 |  |
| $30<t \leq 40$ | 14 |  |
| $40<t \leq 50$ | 4 |  |

(a) Complete the cumulative frequency table.
(b) On the grid, draw a cumulative frequency graph for your table.

(c) Use your graph to find an estimate for the number of calls the operator took more than 18 seconds to answer.
6. 200 students took a test. The cumulative frequency graph gives information about their marks.


The lowest mark scored in the test was 10.
The highest mark scored in the test was 60.
Use this information and the cumulative frequency graph to draw a box plot showing information about the students' marks.

(Total 3 marks)
7. On Friday, Peter went to the airport.

He recorded the number of minutes that each plane was delayed.
He used his results to work out the information in this table.

|  | Minutes |
| :--- | :--- |
| Shortest delay | 0 |
| Lower quartile | 2 |
| Median | 8 |
| Upper quartile | 18 |
| Longest delay | 41 |

(a) On the grid, draw a box plot to show the information in the table.


Peter also went to the airport on Saturday.
He recorded the number of minutes that each plane was delayed.
The box plot below was drawn using this information.

(b) Make two comparisons between the distributions of plane delays on Friday and on Saturday.
$\qquad$
$\qquad$
$\qquad$
$\qquad$

## Histograms

## Things to remember:

- Frequency = Frequency Density x Class Width;
- The y-axis will always be labelled "frequency density";
- The $x$-axis will have a continuous scale.


## Questions:

1. One Monday, Victoria measured the time, in seconds, that individual birds spent on her bird table. She used this information to complete the frequency table.

| Time $(t$ seconds) | Frequency |
| :--- | :--- |
| $0<t \leq 10$ | 8 |
| $10<t \leq 20$ | 16 |
| $20<t \leq 25$ | 15 |
| $25<t \leq 30$ | 12 |
| $30<t \leq 50$ | 6 |

(a) Use the table to complete the histogram.


On Tuesday she conducted a similar survey and drew the following histogram from her results.

(b) Use the histogram for Tuesday to complete the table.

| Time ( $t$ seconds) | Frequency |
| :--- | :--- |
| $0<t \leq 10$ | 10 |
| $10<t \leq 20$ |  |
| $20<t \leq 25$ |  |
| $25<t \leq 30$ |  |
| $30<t \leq 50$ |  |

2. This histogram gives information about the books sold in a bookshop one Saturday.

(a) Use the histogram to complete the table.

| Price $(\boldsymbol{P})$ in pounds $(£)$ | Frequency |
| :--- | :--- |
| $0<P \leq 5$ |  |
| $5<P \leq 10$ |  |
| $10<P \leq 20$ |  |
| $20<P \leq 40$ |  |

The frequency table below gives information about the books sold in a second bookshop on the same Saturday.

| Price $(\boldsymbol{P})$ in pounds $(£)$ | Frequency |
| :--- | :--- |
| $0<P \leq 5$ | 80 |
| $5<P \leq 10$ | 20 |
| $10<P \leq 20$ | 24 |
| $20<P \leq 40$ | 96 |

(b) On the grid below, draw a histogram to represent the information about the books sold in the second bookshop.

3. The incomplete table and histogram give some information about the distances walked by some students in a school in one year.

(a) Use the information in the histogram to complete the frequency table.

| Distance (d) in km | Frequency |
| :--- | :--- |
| $0<d \leq 300$ | 210 |
| $300<d \leq 400$ | 350 |
| $400<d \leq 500$ |  |
| $500<d \leq 1000$ |  |

(b) Use the information in the table to complete the histogram.
4. The incomplete histogram and table show information about the weights of some containers.

| Weight $(w)$ in $\mathbf{k g}$ | Frequenc <br> $\mathbf{y}$ |
| :--- | :--- |
| $0<w \leq 1000$ | 16 |
| $1000<w \leq 2000$ |  |
| $2000<w \leq 4000$ |  |
| $4000<w \leq 6000$ | 16 |
| $6000<w \leq 8000$ |  |
| $8000<w \leq 12000$ | 8 |

(a) Use the information in the histogram to complete the table.
(b) Use the information in the table to complete the histogram.


Weight ( $w$ ) in kg
(2)
(Total 4 marks)
5. The incomplete histogram and table give some information about the distances some teachers travel to school.

(a) Use the information in the histogram to complete the frequency table.

| Distance (dkm) | Frequency |
| :--- | :--- |
| $0<d \leq 5$ | 15 |
| $5<d \leq 10$ | 20 |
| $10<d \leq 20$ |  |
| $20<d \leq 40$ |  |
| $40<d \leq 60$ | 10 |

(b) Use the information in the table to complete the histogram.
6. The table gives information about the heights, in centimetres, of some 15 year old students.

| Height $(h \mathrm{~cm})$ | $145<h \leq 155$ | $155<h \leq 175$ | $175<h \leq 190$ |
| :--- | :--- | :--- | :--- |
| Frequency | 10 | 80 | 24 |

Use the table to draw a histogram.

(Total 3 marks)
7. A teacher asked some year 10 students how long they spent doing homework each night. The histogram was drawn from this information.


Use the histogram to complete the table.

| Time $(t$ <br> minutes $)$ | Frequency |
| :--- | :--- |
| $10 \leq t<15$ | 10 |
| $15 \leq t<30$ |  |
| $30 \leq t<40$ |  |
| $40 \leq t<50$ |  |
| $50 \leq t<70$ |  |

(Total 2 marks)

## Set Theory

Things to remember:


The complement of $\mathbf{A}$ is the region that is not $A$. $A^{\prime}$
This means not A.

## Questions:

1. 

$$
\begin{aligned}
& \mathscr{E}=\{1,2,3,4,5,6,7,8,9,10\} \\
& A=\{\text { multiples of } 2\} \\
& A \cap B=\{2,6\} \\
& A \cup B=\{1,2,3,4,6,8,9,10\}
\end{aligned}
$$

Draw a Venn diagram for this information.
2. Here is a Venn diagram.
(a) Write down the numbers that are in set
(i) $A \cup B$
$\qquad$
(ii) $A \cap B$
$\qquad$
(2)


One of the numbers in the diagram is chosen at random.
(b) Find the probability that the number is in set $A^{\prime}$
3. Sami asked 50 people which drinks they liked from tea, coffee and milk.

All 50 people like at least one of the drinks
19 people like all three drinks.
16 people like tea and coffee but do not like milk.
21 people like coffee and milk.
24 people like tea and milk.
40 people like coffee.
1 person likes only milk.
Sami selects at random one of the 50 people.
(a) Work out the probability that this person likes tea.
$\qquad$
(b) Given that the person selected at random from the 50 people likes tea, find the probability that this person also likes exactly one other drink.

## Proportion

## Things to remember:

- Start by checking the question for squares, cubes and roots;
- " $x$ is directly proportional to $y$ " looks like $\mathbf{x} \boldsymbol{\alpha} \mathbf{y}$ or $\mathbf{x}=\mathbf{k y}$
- " x is inversely proportional to y " looks like $\mathrm{x} \alpha \frac{1}{y}$ or $\mathrm{x}=\frac{k}{y}$


## Questions:

1. The shutter speed, $S$, of a camera varies inversely as the square of the aperture setting, $f$. When $f=8, S=125$
(a) Find a formula for $S$ in terms of $f$.
$\qquad$
(b) Hence, or otherwise, calculate the value of $S$ when $f=4$

$$
S=
$$

2. In a factory, chemical reactions are carried out in spherical containers.

The time, $T$ minutes, the chemical reaction takes is directly proportional to the square of the radius, $R \mathrm{~cm}$, of the spherical container.
When $R=120, T=32$
Find the value of $T$ when $R=150$
$\qquad$
3. $d$ is directly proportional to the square of $t$.
$d=80$ when $t=4$
(a) Express $d$ in terms of $t$.
$\qquad$
(b) Work out the value of $d$ when $t=7$

$$
d=
$$

$\qquad$
(c) Work out the positive value of $t$ when $d=45$

$$
t=
$$

4. The distance, $D$, travelled by a particle is directly proportional to the square of the time, $t$, taken. When $t=40, D=30$
(a) Find a formula for $D$ in terms of $t$.

$$
D=
$$

$\qquad$
(b) Calculate the value of $D$ when $t=64$
(c) Calculate the value of $t$ when $D=12$

Give your answer correct to 3 significant figures.
5. The time, $T$ seconds, it takes a water heater to boil some water is directly proportional to the mass of water, $m \mathrm{~kg}$, in the water heater. When $m=250, T=600$
(a) Find T when $m=400$

$$
\begin{equation*}
T= \tag{3}
\end{equation*}
$$

$\qquad$
The time, $T$ seconds, it takes a water heater to boil a constant mass of water is inversely proportional to the power, $P$ watts, of the water heater.
When $P=1400, T=360$
(b) Find the value of $T$ when $P=900$

$$
T=
$$

6. A ball falls vertically after being dropped.

The ball falls a distance $d$ metres in a time of $t$ seconds.
$d$ is directly proportional to the square of $t$.
The ball falls 20 metres in a time of 2 seconds.
(a) Find a formula for $d$ in terms of $t$.

$$
d=
$$

(b) Calculate the distance the ball falls in 3 seconds.
(c) Calculate the time the ball takes to fall 605 m .
7. In a spring, the tension ( $T$ newtons) is directly proportional to its extension $(x \mathrm{~cm})$. When the tension is 150 newtons, the extension is 6 cm .
(a) Find a formula for $T$ in terms of $x$.

$$
T=
$$

(b) Calculate the tension, in newtons, when the extension is 15 cm .
(c) Calculate the extension, in cm , when the tension is 600 newtons.
8. $f$ is inversely proportional to $d$.

When $d=50, f=256$
Find the value of $f$ when $d=80$

$$
f=
$$

## Percentages - compound interest

## Things to remember:

- New amount $=$ original amount x multiplier ${ }^{n}$


## Questions:

1. Henry invests $£ 4500$ at a compound interest rate of $5 \%$ per annum.

At the end of $n$ complete years the investment has grown to £5469.78.
Find the value of $n$.
2. Bill buys a new machine.

The value of the machine depreciates by $20 \%$ each year.
(a) Bill says 'after 5 years the machine will have no value'.

Bill is wrong. Explain why.
$\qquad$
$\qquad$
$\qquad$
Bill wants to work out the value of the machine after 2 years.
(b) By what single decimal number should Bill multiply the value of the machine when new?
3. Gwen bought a new car. Each year, the value of her car depreciated by $9 \%$.

Calculate the number of years after which the value of her car was $47 \%$ of its value when new.
4. The value of a car depreciates by $35 \%$ each year.

At the end of 2007 the value of the car was $£ 5460$
Work out the value of the car at the end of 2006
5. Toby invested $£ 4500$ for 2 years in a savings account. He was paid $4 \%$ per annum compound interest.
(a) How much did Toby have in his savings account after 2 years?
$£$

Jaspir invested £2400 for $n$ years in a savings account.
He was paid $7.5 \%$ per annum compound interest.
At the end of the $n$ years he had $£ 3445.51$ in the savings account.
(b) Work out the value of $n$.
6. Mario invests $£ 2000$ for 3 years at $5 \%$ per annum compound interest.

Calculate the value of the investment at the end of 3 years.
£

## Percentages - reverse

## Things to remember:

- Work out what the multiplier would have been;



## Questions:

1. Loft insulation reduces annual heating costs by $20 \%$.

After he insulated his loft, Curtley's annual heating cost was $£ 520$.
Work out Curtley's annual heating cost would have been, if he had not insulated his loft.
$£$ $\qquad$
(Total 3 marks)
2. In a sale, normal prices are reduced by $20 \%$.

## SALE - 20\% OFF

Andrew bought a saddle for his horse in the sale.
The sale price of the saddle was £220.
Calculate the normal price of the saddle.
$£$ $\qquad$
(Total 3 marks)
3. Hajra's weekly pay this year is $£ 240$

This is $20 \%$ more than her weekly pay last year.
Bill says 'This means Hajra’s weekly pay last year was $£ 192$ '.
Bill is wrong,
(a) Explain why.
$\qquad$
$\qquad$
(b) Work out Hajra's weekly pay last year.
£
4. The price of all rail season tickets to London increased by $4 \%$.
(a) The price of a rail season ticket from Cambridge to London increased by £121.60 Work out the price before this increase.
£
(b) After the increase, the price of a rail season ticket from Brighton to London was £2828.80
Work out the price before this increase.
£
5. In a sale, normal prices are reduced by $25 \%$.

The sale price of a saw is $£ 12.75$
Calculate the normal price of the saw.
£
(Total 3 marks)
6. In a sale, normal prices are reduced by $12 \%$.

The sale price of a DVD player is $£ 242$.
Work out the normal price of the DVD player.
£ $\qquad$
7. A garage sells cars.

It offers a discount of $20 \%$ off the normal price for cash.
Dave pays $£ 5200$ cash for a car.
Calculate the normal price of the car.

## Useful websites:

www.mathswatchvle.com
www.methodmaths.com
www.hegartymaths.com
www.mymaths.co.uk
www.drfrost.com

## www.bbc.co.uk/schools/gcsebitesize /maths

## Remember: Do your best; it is all you can do ©

